NON-ABELIAN GAUGED CHIRAL BOSON WITH A GENERALIZED FADDEEVIAN REGULARIZATION

Anisur Rahaman[1]
Durgapur Govt. College, Durgapur-713214, Burdwan, West Bengal, INDIA

PACS Nos: 11.15q, 11.10Kk, 11.10Ef

Abstract

We consider non-Abelian gauged version of chiral boson with a generalized Faddeevian regularization. It is a second class constrained theory. We quantize the theory and analyze the phase space. It is shown that in spite of the lack of manifest Lorentz invariance in the action, it has a consistent and Poincare' invariant phase space structure.

[1]e-mail: anisur.rahman@saha.ac.in

Chiral boson has not only acquired a significant position in in the context of string theory but also it is of particular interest in the context of two dimensional anomalous gauge theory [1-10]. Two equivalent description are available for the chiral boson. A manifestly Lorentz invariant formulation was done by Siegel using an auxiliary field [1]. In this formulation it was found that the classical reparametrization invariance of the action did not maintain because of the gravitational anomaly at the quantum mechanical level. An alternative formulation was given by Floreanini and Jackiw [2]. Though it did not suffer from Gravitational anomaly it was lacking Lorentz invariance at the action level however Lorentz invariance was not really broken there. Subsequently a Lorentz invariant formulation was revived by Bellucci, Golterman and Petcher [3].

The interaction of chiral boson with gauge field has also been extensively studied. The vector way of gauging was found to be inconsistent by Bellucci, Golterman and Petcher and by Floreanini and Jackiw [2, 3]. However Chiral way of gauging was found to render consistent anomalous field theory [4, 6, 7].

In Ref [6], Mitra and Ghosh presented a new way of gauging Chiral boson. The lagrangian of this model can be obtained from the Chiral Schwinger model when it is studied with a Faddeevian regularization [7] imposing a chiral constraint [5]. The regularization presented by Mitra was improved by Abreu *etal.* [10] and showed that in place of a specific mass like counter term a one parameter class of mass like counter term of Faddeevian class lead to a consistent anomalous gauge field theory. Recently we have presented a consistent description of the above model in terms of chiral boson [11] in a similar way as Harada did it in his paper [5] for the Chiral Schwinger model with Jackiw-Rajaraman regularization.

In the above models the interacting fields were Abelian type. Various authors tried to generalize the model introducing the non-Abelian gauge field [12, 13, 14, 15, 16, 17]. Non- Abelian generalization of the chiral Schwinger model [12] was appeared immediately after Jackiw and Rajaraman presented the Abelian version of it [4]. And later generalization of both the anomalous and non-anomalous model and also other variants came to the literature [12, 13, 14, 15, 16, 17]. Faddeev in his paper presented a special type of regularization where Gauss law of the theory itself becomes second class. However it would be possible to quantize the theory. Commonly it is known as Faddeevian regularization. Faddeevian class of anomalous gauge theory (regularization) has been becoming a subject of considerable interest for the last

few years [6, 10, 11]. So the non-Abelian generalization of Chiral Schwinger model with a generalized Faddeevian Regularization and description of this model in terms of chiral boson would certainly be a subject of new investigation.

In this letter we have started with a bosonized non-Abelian version of the chiral Schwinger model where Faddeevian class of anomaly appeared because of the presence of generalized regularization proposed by Abreu *etal.* in [10] and reproduce a lagrangian which has one more (chiral) constraint in a similar way as it has been done in [11] for the Abelian case. To be more specific and in a simple language we reproduce a non-Abelian version of the model studied in [11]. We analyze the phases pace and show that it leads to a consistent and physically sensible anomalous gauge theory. We also check the Poincare algebra to ensure the Poincare symmetry because the lagrangian is lacking Lorentz covariance to start with.

The fermionic form of the non-Abelian theory is described by the generating functional

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{i\int d^2x \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/\frac{(1+i\gamma_5)}{2})\psi - \frac{1}{4}tr(F_{\mu\nu}F^{\mu\nu})}, \tag{1}$$

where ψ is the $(1+1)$ dimensional fermionic field furnishing some representation Γ of some simple group G. It interacts with the gauge field through a chiral coupling. A_μ can be written as $A^a_\mu\tau^a$ with τ^a being the the group generators in Γ satisfying $[\tau^a, \tau^b] = -if^{abc}\tau^c$ and $tr(\tau^a\tau^b) = \delta^{ab}$. The non-Abelian field strength is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - e[A_\mu, A_\nu]. \tag{2}$$

We use the convention for the metric $g_{\mu\nu} = diag(+1, -1)$, and for the anti-symmetric tensor $\epsilon^{01} = 1$.

Since in the bosonized formulation the anomaly shows up at the classical level it is the natural setting for the phase space analysis of this theory. In order to get the bosonized action we first integrate the fermions out and construct the effective action. That will yield a determinant of the chiral Dirac operator in presence of the background field. So the partition function becomes

$$Z = \int \mathcal{D}A_\mu \;\; |\det i(D\!\!\!\!/_{ch}[A])| \;\; e^{i\int -\frac{1}{4}tr(F^2)}. \tag{3}$$

The chiral Dirac operator is given by

$$D\!\!\!\!/_{ch} = i\partial\!\!\!/ - eA\!\!\!/\frac{(1+i\gamma_5)}{2}. \tag{4}$$

We write $i\Gamma(A) = i \ln(\det \not{D}_{ch}[A])$ so that the effective action becomes $S_{eff} = \Gamma[A] + \int -\frac{1}{4}F^2$. The contribution of the fermionic loops is contained in the first term of the effective action.

The effective action is not suitable for canonical analysis because of its non-local nature and so is not suitable for canonical analysis. It is due to the $\Gamma[A]$, the loop contribution and so we introduce an auxiliary bosonic field to express it in terms of a local action. This can be done by studying the behavior of $\Gamma[A]$ under gauge transformation [12, 15]. The resulting lagrangian is

$$\begin{aligned}\Gamma[A] &= \frac{1}{8\pi}\int d^2xtr(N_\mu N^\mu) + \frac{1}{12\pi}\int_v d^3y\epsilon^{ijk}tr(N_iN_jN_k) \\ &+ \int d^2xtr[(-\frac{ie}{4\pi}(g^{\mu\nu} + \epsilon^{\mu\nu})N_\mu A_\nu \\ &+ \frac{e^2}{8\pi}[(A_0^2 - A_1^2) - 2\alpha A_1(A_0 + A_1)]. \end{aligned} \tag{5}$$

where $N_\mu = \eta^{-1}\partial_\mu\eta$. We should mention here that unlike the model presented by Harada in [15] this model lacks manifest Lorentz invariance. The first two terms describe the dynamics of the group valued field η and is called the Wess-Zumino action. The first one is the kinetic term and the second one is the Wess-Zumino term which we will denote by S_{wz}. The third term describes its interaction with the gauge field. The last one is a gauge field mass-like term which is not unique because of the ambiguity in the regularization. We have taken the generalization of the term that was used in the Abelian model [6]. In what follows we shall deal with this bosonized action.

In order to investigate the consistency of this theory we proceed towards the hamiltonian analysis of the theory and determine the phases pace structure. It is not difficult to calculate the canonical momenta of the various fields. Before that we express the Wess-Zumino term in the first order form.

$$S_{wz} = \frac{1}{4\pi}\int d^2xtr[\mathcal{A}(\eta)\partial_0\eta], \tag{6}$$

where $\mathcal{A}(\eta)$ is not known but the "magnetic field strength" is given by

$$\begin{aligned}\mathcal{F}_{ij;kl} &= \frac{\partial A_{ij}}{\partial\eta_{lk}} - \frac{\partial A_{kl}}{\partial\eta_{ji}} \\ &= \partial_1\eta_{il}^{-1}(x)\eta_{kj}^{-1}(x) - \partial_1\eta_{kj}^{-1}(x)\eta_{il}^{-1}(x). \end{aligned} \tag{7}$$

The canonical momenta are obtained by taking the derivative of the action with respect to the velocities.

$$\begin{aligned} p_{ij} &= \frac{\delta S}{\delta(\partial_0 \eta_{ij})} = \frac{1}{4\pi}[\partial_0 \eta^{-1}_{ji} + \mathcal{A}(\eta)_{ji} - ie((A_0 - A_1)\eta^{-1})_{ji}], \\ \Pi^{1a} &= F^a_{01}. \end{aligned} \tag{8}$$

For later convenience we define $\tilde{P}_{ij} = P_{ji} - \frac{1}{4\pi}\mathcal{A}(\eta)_{ij}$. The hamiltonian is now obtained in straightforward manner

$$\begin{aligned} H &= \frac{1}{8\pi}\int dx^1 tr(\eta^1 \partial_0 \eta \eta^{-1} \partial_0 \eta + \eta^{-1}\partial_1 \eta \eta^{-1} \partial_1 \eta) \\ &- \frac{ie}{4\pi}\int dx^1 tr \eta^{-1}\partial_1 \eta (A_0 - A_1) + \int dx^1 tr[\frac{1}{2}\Pi_1^2 \\ &- \frac{e^2}{8\pi}(A_0^2 - A_1^2 + 2\alpha A_1(A_0 + A_1)) - (D_1 \Pi^1)^a A_0^a], \end{aligned} \tag{9}$$

where we should replace the $\partial_0 \eta$ in terms of the momenta and have dropped a total derivative term. We can write down the hamiltonian in terms of the free fermionic currents by.

$$\begin{aligned} H_c &= \int dx^1 \mathcal{H}_c \\ &= \int dx \pi(\rho_R^2 + \rho_L^2) + e[\rho_R + \frac{e}{8\pi}(A_0 - A_1)](A_0 - A_1) \\ &\quad + \frac{1}{2}\Pi^2 - \frac{e^2}{8\pi}(A_0^2 - A_1^2 + 2\alpha A_1(A_0 + A_1) - (D_1 \Pi^1) A_0]. \end{aligned} \tag{10}$$

where the fermionic currents are give in terms of canonical variables by

$$\rho_R = \frac{i}{4\pi}(-4\pi \tilde{P} g + g^{-1}\partial_1 g), \qquad \rho_L = \frac{i}{4\pi}(4\pi g \tilde{P} - g \partial_1 g^{-1}), \tag{11}$$

We find that ρ_R and ρ_L satisfy the following Poisson brackets

$$\{\rho^a_{R,L}(x), \rho^b_{R,L}(y)\} = -if^{abc}\rho^c_{R,L}(x)\delta(x-y) \pm \frac{\delta^{ab}}{2\pi}\delta'(x-y), \tag{12}$$

where $\{\rho_R, \rho_L\}$ gives vanishing Poisson brackets. The Poisson brackets of the group valued fields are found out to be

$$\{\eta_{ij}(x), \tilde{P}_{kl}(y)\} = \delta_{ik}\delta_{jl}\delta(x-y), \tag{13}$$

$$\{\tilde{P}_{ij}(x), \tilde{P}_{kl}(y)\} = -\frac{1}{4\pi}\mathcal{F}(\eta)_{ij,kl}\delta(x-y). \tag{14}$$

Let us now introduce the chiral constraint for this non-Abelian version

$$\Omega^a = tr\tau^a\eta\tilde{P}^T + \frac{1}{4\pi}tr\tau^a\partial_1\eta\eta^{-1} \approx 0. \tag{15}$$

If we now impose this constraint in the phases pace of the theory as we have done in the Abelian model [11] we will obtain the hamiltonian in the chiral constraint surface which certainly contains less degrees of freedom.

$$\begin{aligned} H_{CH} &= \frac{1}{4\pi}\int dx^1 tr\eta^{-1}\partial_-\eta\eta^{-1} - \frac{ie}{2\pi}\int dx^1 tr\eta^{-1}\partial_1\eta(A_0 - A_1) \\ &- \frac{e^2}{4\pi}\int dx^1 tr((\alpha-1)A_1^2 + (\alpha+1)A_1A_0) + \frac{1}{2}\int dx^1(\pi^1)^2 \\ &+ \int dx^1(D_1\pi^1)^a A_0^a. \end{aligned} \tag{16}$$

It is straightforward to see that this hamiltonian can be obtained from the action

$$\begin{aligned} I_{CH}[\eta, A] &= \frac{1}{4\pi}\int d^2x tr(\eta^{-1}\partial_1\eta\eta^{-1}\partial_1\eta) + \frac{1}{12\pi}\int d^2x\epsilon^{ijk}N_iN_jN_k \\ &- \frac{ie}{2\pi}\int dx^2 tr\eta^{-1}\partial_1\eta(A_0 - A_1) \\ &+ \frac{e^2}{4\pi}\int d^2x tr[(\alpha-1)A_1^2 + (\alpha+1)A_1A_0] \\ &+ \frac{1}{2}\int d^2x trF_{01}^2. \end{aligned} \tag{17}$$

It is the non-Abelian generalization of the action obtained in [11]. We would also like to mention that it is the gauged version of the action for the non-Abelian chiral boson with a generalized Faddeevian regularization. In contrast to the model presented by Harada [15] this action too lacks manifest Lorentz covariance. It is fair to say that the imposition of the chiral constraint in this fashion is not new in field theory. In $(1+1)$ dimensional field theory Harada is the first person to used this [5]. However the scope to use this formalism is very limited. In that sense it is interesting to see the use of it once more in the non-Abelian case.

Our next task is to quantize the theory. We find that the two primary constraints of the theory are

$$\omega_1^a = \pi_0^a = 0, \tag{18}$$

$$\tilde{P}_{ij} - \frac{1}{4\pi}\partial_1\eta_{ij} = 0. \tag{19}$$

The second constraint in (19) reproduces the chiral constraint (15). It is convenient to write the above constraint multiplying by η from the left.

$$(\omega^2)_{ij} = \eta\tilde{P}^T + \frac{1}{4\pi}(\partial_1\eta\eta^{-1})_{ij} = 0, \tag{20}$$

which is equivalent to $\rho_L^a = 0$. It is a second class constraint itself having the following Poisson brackets between themselves in two different points

$$\{\omega_{ij}^2(x), \omega_{kl}^2(y)\} = \frac{1}{4\pi}\epsilon_{ij}\delta_{jk}.\delta(x-y) \tag{21}$$

In order to get the full constraint structure we adjoin the primary constraints with velocities to the canonical hamiltonian given in (16) and the total (effective) hamiltonian turns out to be

$$\mathcal{H} = \mathcal{H}_c + u^a\rho_L^a + v^a\pi_0^a. \tag{22}$$

where u^a and v^a are velocities corresponding to the primary constraints The two primary constraints should preserve in time in order to maintain consistency of the theory and that leads to two secondary constraints

$$\omega_3^a = (D_1\pi^1)^a + e\rho_R^a + (1+\alpha)\frac{e^2}{4\pi}A_1^a = 0, \tag{23}$$

$$\omega_4^a = (1+\alpha)\pi^{1a} + 2\alpha(A_0 + A_1)' = 0, \tag{24}$$

and the velocity u^a is found out to be

$$u^a = -i\eta(\eta^{-1}\partial_1\eta - ieA_-)\eta^{-1}. \tag{25}$$

ω_3^a is known as the gauss law constraint of the theory. The Poisson brackets between the different constraints of the theory are

$$\{\omega_1^a(x), \omega_4^b(y)\} = 2\alpha\delta'(x-y), \tag{26}$$

$$\{\omega_4^a(x), \omega_4^b(y)\} = 4\alpha(1+\alpha)\delta'(x-y), \tag{27}$$

$$\{\omega_3^a(x), \omega_4^b(y)\} = 2\alpha D^{ab}\delta'(x-y) + (1+\alpha)\frac{e^2}{4\pi}\delta(x-y), \tag{28}$$

$$\{\omega_3^a(x), \omega_3^b(y)\} = ef^{abc}[\omega_3^c + \frac{e^2}{4\pi}(1+\alpha)A_1^c] + \frac{e^2}{2\pi}\alpha\delta^{ab}\delta'(x-y). \tag{29}$$

where $D^{ab}\delta(x-y) = [-\partial_1^a\delta^{ab} + if^{abc}A_1^c(x)]\delta(x-y)$. Other Poisson brackets give vanishing values. It is now straightforward to see that the Dirac brackets [18] of the fields describing the hamiltonian in the constraint surface is

$$[A_1^a(x), A_1^b(y)]^* = \frac{2\pi}{e^2}\delta^{ab}\delta'(x-y), \tag{30}$$

$$[A_1^a(x), \pi_1^b(y)]^* = \frac{(\alpha-1)}{2\alpha}\delta^{ab}\delta(x-y), \tag{31}$$

$$[\pi_1^a(x), \pi_1^b(y)]^* = \frac{(\alpha+1)^2}{16\alpha\pi}\delta^{ab}\epsilon(x-y). \tag{32}$$

We can now impose the constraints of the theory strongly into the hamiltonian and find out the hamiltonian on the constrained surface which will be consistent with the Dirac brackets. The hamiltonian density in the reduced space is

$$\mathcal{H}_R = \frac{\pi}{e^2}(D_1\pi^1)^2 + \frac{1}{2}(1-\alpha)(D_1\pi^1)^a A^{1a} + \frac{e^2}{16\pi}[(1+\alpha)^2 - 8\alpha]A_1^2. \tag{33}$$

Seeing the bosonized lagrangian one may think that the Lorentz non-invariant counter term may spoil the explicit Lorentz invariance of the lagrangian. However, a closer look reveals the invariance of the theory on the physical subspace. The fact that it is maintained only on the physical subspace instead of the whole phase space is because of its deceptive appearance. In order to show the invariance we have to demonstrate the validity of the Poincare' algebra of this two dimensional system. There are three elements in the algebra, the hamiltonian H, the momentum P and the boost generator M which have to satisfy the relation

$$\{P, H\} = 0, \qquad \{M, P\} = -H, \qquad \{M, H\} = -P, \tag{34}$$

to ensure the Poincare' invariance of a theory. The hamiltonian density has already been evaluated in (33). The momentum density can be written, by discarding a total derivative, as

$$\begin{aligned}\mathcal{P}(x) &= tr\pi^{1a}\partial_1 A_1^a + tr\tilde{P}^T\partial_1\eta \\ &= \pi^{1a}A_1^a + \frac{\pi}{e^2}[(D_1\pi^1)^a + \frac{(1+\alpha)e^2}{4\pi}A_1^a]^2. \end{aligned} \tag{35}$$

The boost generator can be expressed in terms of the hamiltonian and the momentum densities as

$$M = tP + \int dx x\mathcal{H}_R(x). \tag{36}$$

From the above expression one can evaluate the left hand sides of (34). One should use the Dirac brackets and that involves a tedious calculation. However, there arises a simplification due to the fact that the brackets between the different terms in the densities are all canonical except the terms appeared as $\{A_1(x), A_1(y)\}$ and $\{\pi^1(x), \pi^1(y)\}$. The rest is a straightforward calculation of canonical brackets which yields

$$\{\mathcal{H}_R(x), p(y)\} = \partial_1\mathcal{H}_R(x)\delta(x-y)+.., \quad \{\mathcal{H}_R(x), \mathcal{H}_R(y)\} = \partial_1 p(x)\delta(x-y)+.., \tag{37}$$

where the dots represent some $\delta'(x-y)$ terms. Now substituting this expressions one can easily check the Poincare' algebra (34). This implicit invariance of the theory suggests that there may exist a manifestly Lorentz invariant form of lagrangian [3].

So the non-Abelian generalization of the chiral Schwinger model with new Faddeevian regularization [10, 11], as shown above, has a consistent and Lorentz invariant hamiltonian structure. The unitarity of the model is not obvious and one has to go through a BRST analysis for formal proof of unitarity. However we can expect that the theory will respect the unitarity since QCD_2 is a super renormalizable theory. The unraveling of its physical properties would be the next important task. But unfortunately, unlike its Abelian ancestor, it does not have exact solvability. Further, the absence of the gauge invariance does not allow us to use the simplifications associated with the light-cone gauge that occurs in vector QCD_2 [19]. The gauge invariance however can be recovered by going to its gauge invariant formulation. But the price to be paid is the extra Wess-Zumino field. This

formulation can help to study the existence of the mesonic bound states, and it may also help to study the different regimes of a theory that are associated with the different extremes of the coupling and the relation between them. The non-Abelian duality of this model [19] which involves writing down of a theory in terms of some other fields with the coupling constant inverted is also another interesting thing to be considered seriously. Such studies may help the understanding of non-perturbative physics.

It is a pleasure to thank Prof. P. Mitra for useful discussions. I also want to thank the Director and the Head of the Theory Division of Saha Institute of Nuclear Physics, Kolkata for providing computer facilities.

References

[1] W. Siegel, Nucl. Phys. **238** 307 (1984)

[2] R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59** 1837 (1987)

[3] S. Bellucci, M. F. L. Golterman and D. N. Petcher, Nucl. Phys. **B326** 307 (1989)

[4] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54** 1219 (1985); Phys. Lett. **193B** (1987) 257

[5] K. Harada, Phys. Rev. Lett. **64** 139 (1990)

[6] S. Ghosh and P. Mitra Phys. Rev. **D44** 1332 (1991); Phys. Lett. **B284** 23 (1992)

[7] L. D. Faddeev, Theo. Math. Phys. **1** (1969) 1

[8] Imbimbo and S. Schwimmer, Phys. Lett. **B193** 455 (1987)

[9] J. M. F. Labstida and M. Permici, Nucl. Phys. **B297** 557 (1988)

[10] E. C .M. Abreu, A. Ilha, C. Nevs and C. Wotzasek, Phys. Rev. **D61** 025014 (1999)

[11] A. Rahaman Hep-th 0306037, to appear in Int. Jour. Mod. Phys. Vol. 21 2006.

[12] R. Rajaraman, Phys. Lett. **B162** 148 (1985)

[13] L. D. Faddeev, Phys. Lett. **B145** 81 (1984); L. D. Faddeev and S. L. Satashvilli, Phys. Lett. **B167** 225 (1986).

[14] E. Abdalla, K. D. Rothe, Phys. Rev. **D36** 3190 (1987)

[15] K. Harada, Int. Jour. Mod. Phys. **A5** 4469 (1990); Nucl. Phys. **B329** 723 (1990)

[16] A. Rahaman and S. Mukhopadhyay, Mod. Phys. Lett. **A12** 3121 (1997)

[17] K. Harada and I. Tsutsui, Phys. lett. **183B** 311 (1987)

[18] P. A. M. Dirac, *Lectures on quantum mechanics*, Belfer Graduate School of Science, Yeshiva University, New York, 1964.

[19] E. Abdalla and M. C. B. Abdalla, *Updating* QCD_2, hep-th/9503002, Phys. Rept. **256** 253 (1996).